NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 7
COURSE CODE: RAN701S	**COURSE NAME:** REAL ANALYSIS
SESSION: JUNE 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	DR. NA CHERE
MODERATOR:	PROF. F MASSAMBA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. Number the answers clearly.
4. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [11]

Let (x_n) be a sequence of real numbers and $x \in \mathbb{R}$.

1.1. Define what does it mean to say the sequence (x_n) converges to x? [2]

1.2. Use the definition in part (1.1) to establish the sequence $\left(\frac{n-3n^2}{n^2+2n}\right)$ converges to -3. [9]

QUESTION 2 [13]

Determine whether each of the following sequences is convergent or divergent.

2.1. $\left((n+1)^{\frac{1}{\ln(n+1)}}\right)$. [7]

2.2. $\left(1-(-1)^n+\frac{1}{n}\right)$. [6]

QUESTION 3 [10]

3.1. Define what does it mean to say a sequence (x_n) in $\mathbb{R}$ is bounded? [3]

3.2. Prove that if (x_n) is convergent then it is bounded. [7]

Question 4 [13]

4.1. Define what does it mean to say a sequence (x_n) in $\mathbb{R}$ is a Cauchy sequence? [3]

4.2. Show that the sequence $\left(\frac{2n-2}{n}\right)$ is a Cauchy sequence. [10]

QUESTION 5 [16]

5.1. Find the sum of the series $\sum_{n=0}^{\infty}\frac{2}{(n+2)(n+3)}$, if it converges. [8]

5.2. Determine whether the series $\sum_{n=0}^{\infty}\frac{(-1)^n4^n n^2}{n!}$ converges absolutely or conditionally. [8]

QUESTION 6 [13]

Use the Epsilon- delta (ϵ, δ) definition to show that $\lim_{x\to 1}\frac{3x+5}{x+3} = 2$.

QUESTION 7 [16]

7.1. Use the definition of uniform continuity to show that the function $f(x) = \frac{1}{x+1}$ is uniformly continuous on $[0, \infty)$. [9]

7.2. Use the nonuniform continuity criterion to show that the function $f(x) = \sin\left(\frac{1}{x}\right)$ is not uniformly continuous on $(0, \infty)$. [7]

QUESTION 8 [8]

Apply the mean value theorem to prove that $|\tan y - \tan x| \leq 2|y - x|$ for $x < y$ and $x, y \in \left[0, \frac{\pi}{4}\right]$.

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER